The Multibody Systems Approach to Vehicle Dynamics

A Short Course

Lecture 5 – Tyre Modelling

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Course Agenda

Day 1
Lecture 1 – Introduction to Vehicle Dynamics
Lecture 2 – Multibody Systems Simulation Software

Day 2
Lecture 3 – Modelling and Analysis of Suspension Systems
Lecture 4 – Tyre Characteristics
Lecture 5 – Tyre Modelling

Day 3
Lecture 6 – Modelling and Analysis of the Full Vehicle
Discussion and Wrap Up
Close
Lecture 5 – Tyre Modelling

• History and Tyre Construction
• Tyre Force and Moment Generation
• Tyre Models for Handling and Durability (MF Tyre Model, Finite Element models, Ftire)
• Aircraft Tyre Modelling
• New Developments
History of Tyres

The first pneumatic tyre, 1845 by Robert William Thomson. A number of air filled tube inside a leather case. Resistant to punctures but old solid rubber remained in favour with the public.

http://www.blackcircles.com/general/history

Boyd Dunlop reinvented the pneumatic tyre whilst trying to improve his sons bike in 1888

http://www.blackcircles.com/general/history

In 1895 the pneumatic tyre was first used on automobiles, by Andre and Edouard Michelin.

http://www.blackcircles.com/general/history

Michelin first introduced steel-belted radial tires in Europe in 1948


Michelin first announced the Tweel in 2005, a non-pneumatic tyre, removing the necessity checking tyre pressures

http://auto.howstuffworks.com/tweel-airless-tire.htm
Tyre Sidewall Information

205/55 R15 87V
205 Nominal Section-width in mm

TUBELESS
Tubeless / Tube Type

E4
All passenger car tyres from current production comply with ECE Standard 30

026504
Approval number acc. To ECE regulation 30

4008
Production code (40th week, 2008)

DOT
Department of Transportation, USA

TWI
Tread Wear Indicator (1.6mm)

Continental Technical Data Book. Car Tyres 1999
Tyres for All Seasons

• Motorsport
  - Slicks
  - Intermediates
  - Full Wets

• Passenger Cars
  - Summer Tyres
  - All Season Tyres
  - Winter Tyres
  - Mud and Snow Tyres
Radial Tyre Structure

http://www.dunlopatl.co.uk/tech_support/aircraft-tyre-technology.aspx

Rubber Compound Formulation

- Two Major Ingredients – the rubber and the filler (carbon black and silica)
- Combined to achieve different objectives – maximise wet and dry traction/achieve superior rolling resistance
- Four major types of rubber
  - natural rubber
  - styrene-butadiene rubber (SBR)
  - polybutadiene rubber (BR)
  - butyl rubber (along with halogenated butyl rubber)
- The first three are primarily used as tread and sidewall compounds, while butyl rubber and halogenated butyl rubber are primarily used for the inner liner
- Other ingredients also come into play to aid in the processing of the tire or to function as anti-oxidants, anti-ozonants, and anti-aging agents. In addition, the “cure package”—a combination of curatives and accelerators—is used to form the tire and give it its elasticity.
Tyre Thread Design

- **Groove**
- **Pitch**
- **Rib Shape**
- **Lug Shape**
- **Rib-Lug Shape**

- **Studs**: Further improvement of grip in icy conditions

- **Sipe**: Fine groove in the tread pattern to improve grip in icy conditions
  [http://www.ctyres.co.uk/tyre_info/tyre_type.html](http://www.ctyres.co.uk/tyre_info/tyre_type.html)

- **Block Shape**
- **Asymmetric pattern**
- **Directional pattern**

FEA Tyre Models

http://www.cosin.eu/
Automotive Applications

Main applications are in the automotive industry for:
- Simulation of vehicle handling
- Prediction of vehicle ride quality
- Determination of component loads from systems model
Aerospace Applications

There is a growing need in the aerospace industry for:

- Prediction of shimmy
- Rough field performance
- Ride & controllability
- Ground loads
- Landing, Takeoff, Taxiing
SAE Tyre Axis System

Angular Velocity (ω)
Wheel Torque (T)
Spin Axis

Direction of Wheel Heading
Tractive Force ($F_x$)

Direction of Wheel Travel

Normal Force ($F_z$)

Lateral Force ($F_y$)
Tyre Testing

- Lateral force with slip/camber angle
- Aligning moment with slip/camber angle
- Longitudinal force with slip ratio

Courtesy of Dunlop TYRES Ltd.
Lateral Force $F_y$ with Slip Angle $\alpha$

Courtesy of Dunlop Tyres Ltd.
Tyre Modelling

Prediction of Vehicle Ride Quality
- Simple Physical Models (Stiffness/Damping)
- More Advanced Physical Models (FTire)

Simulation of Vehicle Handling
- Interpolation models (Lookup Tables)
- Simple Equation based representations (Fiala)
- Complex Mathematical Fits to Test Data (Magic Form)
- Pure and Combined Slip Models

Determination of Component Loading
- Simple Physical Models (Equivalent Volume)
- More Advanced Physical Models (FTire)
- Full Non-Linear Finite Element Models
Vehicle/Tyre Model Interaction

**VEHICLE MODEL**
- Wheel centre - Position, Orientation and Velocities
  - Mathematical Solution at Integration Time Steps

**TYRE MODEL**
- $F_x$ - longitudinal tractive or braking force
- $F_y$ - lateral cornering force
- $F_z$ - vertical normal force
- $M_z$ - aligning moment
- $M_x$ - overturning moment
- $M_y$ - rolling resistance moment
Tyre Model/Data Assessment

- FIALA MODEL
- MAGIC FORMULA MODELS
- INTERPOLATION MODELS

Check plots in ADAMS tyre rig model

Vehicle Model

$F_y$

Slip Angle $\alpha$
Tyre Model/Data Assessment
# Tyre Model Summary

<table>
<thead>
<tr>
<th>Application</th>
<th>Tyre Model</th>
<th>Model Type</th>
<th>Specifically for Automotive</th>
<th>Adapted for Aircraft</th>
<th>Specifically for Aircraft</th>
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<tr>
<td>Durability / Vehicle Handling Studies</td>
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<td>ADAMS 5.21</td>
<td>Interpolation</td>
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</table>
Tyre Model Summary

Interpolation Model
• It uses tyre test data to interpolate results for a particular scenario

Empirical Model
• Empirical models use test data to determine equations and parameters to replicate the tyre data

Physical Model
• These use physical representations of tyre to generate results (e.g. radial belts with spring elements)
The Fiala Tyre Model

• **Input Parameters**

![Tyre Dimensions](image1)

![Model Geometry](image2)
The Fiala Tyre Model

Input Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
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<tbody>
<tr>
<td>R1</td>
<td>The unloaded tyre radius</td>
</tr>
<tr>
<td>R2</td>
<td>The tyre carcass radius</td>
</tr>
<tr>
<td>kz</td>
<td>The tyre radial stiffness</td>
</tr>
<tr>
<td>Cs</td>
<td>The longitudinal tyre stiffness</td>
</tr>
<tr>
<td>Ca</td>
<td>Lateral tyre stiffness due to slip angle</td>
</tr>
<tr>
<td>Cg</td>
<td>Lateral tyre stiffness due to camber angle (Not Used)</td>
</tr>
<tr>
<td>Cr</td>
<td>The rolling resistant moment coefficient</td>
</tr>
<tr>
<td>ζ</td>
<td>The radial damping ratio</td>
</tr>
<tr>
<td>m0</td>
<td>The tyre to road coefficient of “static” friction</td>
</tr>
<tr>
<td>m1</td>
<td>The tyre to road coefficient of “sliding” friction</td>
</tr>
</tbody>
</table>
The Fiala Tyre Model
(Lateral Force and Aligning Moment)

• Example of the Lateral Force Generation

The critical slip angle is: \[ \alpha^* = \tan^{-1}\left| \frac{3 \mu F_z}{C_\alpha} \right| \]

And so if |\alpha|<\alpha* then:
\[ F_y = -\mu |F_z|(1 - H^3)\text{sgn}(\alpha), \text{ where } H = 1 - \left( \frac{C_\alpha |\tan\alpha|}{3 \mu |F_z|} \right) \]

Else if |\alpha|>\alpha* then:
\[ F_y = -\mu |F_z| \text{sgn}(\alpha) \]

• Example of the Aligning Moment Generation

And so if |\alpha|<\alpha* then:
\[ M_z = 2 \mu |F_z| R \left(1 - H\right) H^3 \text{sgn}(\alpha) \]

Else if |\alpha|>\alpha* then:
\[ M_z = 0.0 \]
The Fiala Tyre Model

• Limitations
  • No camber thrust
  • Not suitable for combined slip
  • Constant cornering stiffness with load
  • Zero aligning moment at high slip angles
The “Magic Formula” Tyre Model

The basis of this established model is that tyre force and moment curves look like sine functions which have been modified by introducing an arctangent function to “stretch” the slip values on the x-axis.


The “Magic Formula” Tyre Model

The general form of the model (version 3) is:

\[ y(x) = D \sin \left[ C \arctan\{ Bx - E \left( Bx - \arctan( Bx )\right)\} \right] \]

where

\[ Y(X) = y(x) + S_v \]
\[ x = X + S_h \]
\[ S_h = \text{horizontal shift} \]
\[ S_v = \text{vertical shift} \]

\[ Y = F_x, F_y, \text{ or } M_z \]
\[ X = \alpha \text{ or } \kappa \]
The “Magic Formula” Tyre Model

The Main Parameters in the General Equation are

D - is the peak value.

C - is a shape factor that controls the “stretching” in the x direction.
   1.30 - lateral force curve.
   1.65 - longitudinal braking force curve.
   2.40 - aligning moment curve.

B - is referred to as a “stiffness” factor. BCD is the slope at zero slip.

E - is a “curvature” factor which effects the transition in the curve and the position $x_m$ at which the peak value if present occurs. E is calculated using:

$$E = B_x m - \arctan \left( \frac{B_x m}{2C} \right)$$

$y_s$ - is the asymptotic value at large slip values and is found using:

$$y_s = D \sin \left( \frac{pC}{2} \right)$$
At zero camber the cornering stiffness $B_{CDy}$ reaches a maximum value defined by the coefficient $a_3$ at a given value of vertical load $F_z$ which equates to the coefficient $a_4$. The slope at zero vertical load is taken as $2a_3/a_4$. 

$$\arctan \left( \frac{2a_3}{a_4} \right)$$

$$B_{CDy} \quad (N/\text{rad})$$

$$a_4 \quad F_z \quad (N)$$

$0 \quad a_3 \quad \arctan \left( \frac{2a_3}{a_4} \right)$
The “Magic Formula” Tyre Model

- **General Formula (Version 3)**
  \[ y(x) = D \sin \left[ \arctan \left( Bx - E \left( Bx - \arctan (Bx) \right) \right) \right] \]
- **Y(X) = y(x) + Sv**
- **x = X + Sh**
- **B = stiffness factor**
- **C = shape factor**
- **D = peak factor**
- **Sh = horizontal shift**
- **Sh = vertical shift**
- **B = \frac{dy}{dx}(x=0) / CD**
- **C = \frac{2}{p} \arcsin \left( \frac{ys}{D} \right)**
- **D = ymax**
- **E = \frac{(Bxm - \tan(p/2C))/(Bxm - \arctan (Bxm))}{(Bxm)}**

- **Lateral Force**
- **Xy = a**
- **Yy = Fy**
- **Dy = my Fz**
- **my = (a1Fz + a2) \left( 1 - a15 g2 \right)**
- **BCDy = a3 \sin(2 \arctan(Fz/a4)) \left( 1 - a15|g| \right)**
- **Cy = a0**
- **Ey = (a6Fz+a7) \left( 1-(a16g + a17) \sgn(a + Shy) \right)**
- **By = BCDy / CyDy**
- **Shy = a8Fz + a9 + a10g**
- **Svy = a11Fz + a12 + (a13Fz2 + a14Fz)g**
The “Magic Formula” Tyre Model

• Longitudinal Force
  • $X_x = k$
  • $Y_x = F_x$
  • $D_x = m_x F_z$
  • $m_x = b_1 F_z + b_2$
  • $B C D_x = (b_3 F_z^2 + b_4 F_z) \exp(-b_5 F_z)$
  • $C_x = b_0$
  • $E_x = (b_6 F_z^2 + b_7 F_z + b_8) (1 - b_13 \text{sgn}(k + Sh_x))$
  • $B_x = B C D_x / C_x D_x$
  • $S_h x = b_9 F_z + b_{10}$
  • $S_v y = b_{11} F_z + b_{12}$
  • Brake force only ($b_{11} = b_{12} = b_{13} = 0$)

• Aligning Moment
  • $X_z = a$
  • $Y_z = M_z$
  • $D_z = (c_1 F_z^2 + c_2 F_z) (1 - c_{18} g^2)$
  • $B C D_z = (c_3 F_z^2 + c_4 F_z) (1 - c_6 | g |) \exp(-c_{5} F_z)$
  • $C_z = c_0$
  • $E_z = (c_z F_z^2 + c_8 F_z + c_0) (1 - (c_{19} g + c_{20})^*)$
  • $B_z = B C D_z / C_z D_z$
  • $S_h z = c_{11} F_z + c_{12} + c_{13} g$
  • $S_v z = c_{14} F_z + c_{15} + (c_{16} F_z^2 + c_{17} F_z) g$
Laboratory Simulation
- Shaker Rig

RIDE AND VIBRATION

Simple Spring Damper Model
Flexible Ring Method
Modal FE Model in MBS
Ride Simulation (Comfort)

Vehicle Body or Sprung Mass
Suspension Spring and Damper

Also important in Motorsport Vehicles
Also important in Military Vehicles
Component Load

Tyre Modelling

Radial Spring Models
Equivalent Plane Method
Equivalent Volume Method
Flexible Ring Method
Coupled Explicit FE and MBS Methods
Tyre Modelling

FINITE ELEMENT MODELS

A Physical Tyre Model is required
Finite Element Modelling is an example
Can model deformable terrain as well
Not practical in terms of modelling time
Excessive computational effort
Mainly a tool for Tyre Manufacturers
Contact Forces

- Used on Helisafe EU project and for tracked vehicles
- Modelled hard non-destructive landings
- Bi-linear model with tyre and wheel stiffness
- Contact function based on stiffness and damping
- Includes friction

Eurocopter Model 5m/s ground impact
Tracked Vehicle operating in deep snow
Examples of Tyre Interactions
Durability Tyre Models

- Early durability tyre models were 2D
- Radial Spring or Equivalent Plane
- Capture deformed shape of tyre enveloping terrain
- Resultant force towards wheel centre
Durability Tyre Model

- Originally developed in Finland for logging vehicles
- Captured tyre interaction with sawn tree trunks on rough terrain
- 3d Model - Discritization into cross-sectional elements

Definition of tyre carcass shape for a durability tyre model
Early Road/Terrain Model

Road modelled using triangular planar elements
Friction can vary on element by element basis

Intersection of tyre sectional elements with road elements
The FTIRE Model

A Tyre Model for Ride & Durability Simulations
A Flexible ring tyre model
Tire phenomena based on a mechanical model

Developed by Cosin (www.cosin.eu)
The FTIRE Model

- Structural dynamics based, full 3D nonlinear in-plane and out-of-plane tire model for simulation of belt dynamics, local pressure distribution in the contact patch, rolling resistance, side-wall contact, large camber angles and misuse scenarios.
- Suitable for a frequency range up to 200Hz, excited by short surface wavelength, mass imbalance, non-uniformity or irregular tread patterns.
- Very fast and flexible. Orders of magnitude faster than explicit FE models.
- Simulation of imbalances by inhomogeneous mass distribution and local wear.
- Belt temperature distribution model.
The FTIRE Model

Tyre structure described with distributed mass, connected to rim by distributed stiffness & damping elements
3D Road/Terrain Model

Open Source software developed by Daimler AG VIRES GmbH
3D Road Data Curved regular Grid (CRG) Representation
Data Files can be generated from laser scans along a road

Belgian Block XYZ map
Open CRG Visualisation

Regular Grid Road Data Files (RGR Files)
FTire Animations

Detailed Tread Pattern

Truck Tire Passing a big bump
FTire Animations

Rolling over a high kerb

Local Belt Deformation
FTire Animations

Sine wave with decreasing wave length

FTire on RGR Road Surface
FTire: Running over an Obstacle

\[ v = 80 \text{ km/h} \]

wheel load [N]  
longitudinal force [N]
FTire: Handling Characteristics

\[ F_z = 2, 4, 6, 8 \text{ kN} \]
Aircraft Tyre Modelling

- EPSRC Project with AIRBUS UK
- Simulate landing, takeoff, taxiing
- Tyre Testing by Airbus in Toulouse
- Developed model in a MATLAB/SIMULINK Environment with export to ADAMS
- Low Parameter Model based on Harty Approach with extended Load/Speed dependence
- Shimmy Modelling (Early NASA work) remains elusive
Aircraft Tyres

- Aircraft tyres are pushed to the extremes of operational envelopes compared to other forms of ground vehicles.

- Typical automotive applications would see tyres experiencing slip angles of around 10° however aircraft tyres can go up to 90°.

- Typically speeds of up to 200mph will be experienced (225-235mph being the speed ratings typically seen (from Bridgestone (2011))))
Extreme Operating Conditions

- Because of the operational loads of the aircraft, the vertical loads experienced by the tyre’s are also magnified.

- The rated load of a nose wheel can be approx. 40 kN; an automotive tyre of similar size can be seen to have a load of approx. 6 kN.

- Large landing gear tyres can have rated loads in excess of 300 kN.

Daugherty (2003)
Tyre Costs

• The costs involved with aircraft tyres can best be seen when comparing them against an automotive example
• An average automotive tyre can cost around £60-70 (using an example tyre size of 205/55 R16)
• An aircraft tyre of similar size can cost £1,708 (based upon a 27x7.7in (686x196mm) tyre)
• Then comparing this to a large aircraft tyre of size 1,400 x 530mm which costs £4,154 (this is the size of tyre on the main landing gear for an Airbus A380)

(All figures are based upon Michelin prices (from Air Michelin (2007)))
How Fast They Wear

• The rate at which a tyre will wear is highly dependent on the operational life of the tyre
• A tyre can last for days or months depending upon the scenarios it is put through
• If the aircraft lasts for 30 years that can mean an average tyre change of around 500 or more
• Bias ply tyres can be retreaded more than radial (bias ply on average about seven and radials about three retreads)
• This has obvious impacts on the market trends of using radial tyres for weight saving purposes

(Information from an Interview with Dunlop Aircraft Tyres Chairman and Managing Director (from TyrePress (2011)))
Aircraft Tyre Testing

• To test aircraft tyres to the extremes highlighted through the operational characteristics means a specialised array of test facilities is required
• Different facilities have different capacities to explore the responses of the tyres
• These tend to fall into three main brackets:
  1. High straight-line speed (with limited slip angle)
  2. Vertical load (with limited slip angle)
  3. High slip angle (with limited speed)
NASA’s test facility has the capacity to test tyres up to 253mph and slip angles of 15°

(Data from Daugherty (2003))

An example of a dynamometer from MTS working with Boeing

These facilities can apply vertical loads to the tyre as required operationally and can evaluate slip angles to 20-30°

(Data from MTS (2006))

Airbus’s TERATYRE facility is able to examine high slip angles and loads but has speed limitations

(Data from Ding (2006))
The Current Market Trends to 2030

• Between 2000 and 2010 air travel has seen an increase of around 45% despite the numerous events which have affected the market.

• Two of the biggest manufactures have clear views on where the market is heading over the next twenty years to 2030 and they see the same kind of growth again.

• Boeing and Airbus both predicted significant rises in the number of airliners that will be required for the 2030 market.

• Airbus estimates around 29,000 new airliners will be required with Boeing supporting the figures predicting 33,000.

Data in this slide has come from Airbus (2011a) and Boeing (2011a)
Sustainable mobility

Fuel economy improvement by low rolling resistance and constant tyre pressure monitoring

Fuel efficiency labels

Constant tyre pressure monitor using pressure sensors or comparing wheel speeds using ESP

http://www.dunlop.eu/dunlop_uk/what_sets_dunlop_apart/future_eu_tyre_label
http://www.mysheriff.co.uk
http://www.tirepressuremonitoring.com/

Intelligent tyres / Cyber tyre

Conclusions

• A single tyre model for all applications does not currently exist (Ftire?)
• Tyre models are developed to address specific analyses
• Tyre dynamics are complex and highly non-linear
• Tyre models are only as good as the data supplied
• What does the future hold – Rolling Resistance, Tyre Data Monitoring Systems, ...